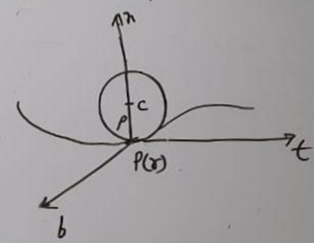
Osculating Circle:

Osculating Circle is the circle which has three point contact with the curve at P.



Let C is the centre of circle with radius $PC = \rho$ and $P(r)$ is any any point on curve then

$$\vec{PC} = \text{Position vector of } C - \text{Position vector of } P$$

$$\Rightarrow \vec{PC} = c - r$$

Again $\vec{PC} = \rho n$

$$\Rightarrow c - r = \rho n$$

$$\Rightarrow \boxed{c = r + \rho n}$$ This is the eqⁿ of osculating circle.

Since $(c-r) = \rho n$

Squaring $(c-r)^2 = \rho^2$

Let $F(s) = (c-r)^2 - \rho^2$ ——— (1)

then for three point contact

$F(s) = 0,\quad F'(s) = 0,\quad F''(s) = 0,$ and $F'''(s) \neq 0$

From (1) $F'(s) = 2(c-r)\cdot\left(-\frac{dr}{ds}\right)$

$$= 2(c-r)\cdot(-t) = 0$$

but $-2 \neq 0$ $\boxed{(c-r)\cdot t = 0}$ — (2)

Again diff. (2) $F''(s) = (c-r)\cdot t' + (-t)\cdot t = 0$

$$= (c-r)\cdot Kn - 1 = 0$$

$$= (c-r)\cdot n = \frac{1}{K}$$

but $(c-r) = \rho n$

$$\Rightarrow \rho n \cdot n = \frac{1}{K}$$

$$\Rightarrow \rho(n\cdot n) = \frac{1}{K}$$

$$\Rightarrow \rho = \frac{1}{K}$$